CHAPTER 2 - MATHEMATICS AND MEASUREMENT: THE TOOLS OF SCIENCE

Why should Christians bother to learn about mathematics and measurement? Many Bible verses hint that these may sometimes be important. For instance:

- "Thou shalt not have in thy bag divers weights, a great and a small. Thou shalt not have in thine house divers measures, a great and a small. But thou shalt have a perfect and just weight, a perfect and just measure shalt thou have: that thy days may be lengthened in the land which the LORD thy God giveth thee." Deut 25:13-15
- "And he said unto them, Take heed what ye hear: with what measure ye mete, it shall be measured to you: and unto you that hear shall more be given." Mark 4:24 (KJV)
- Regarding wooden idols: "The carpenter stretcheth out his rule; he marketh it out with a line; he fitteth it with planes, and he marketh it out with the compass." Isaiah 44:13
- "Ye shall have just balances, and a just ephah, and a just bath. The ephah and the bath shall be of one measure, that the bath may contain the tenth part of an homer, and the ephah the tenth part of an homer: the measure thereof shall be after the homer. And the shekel shall be twenty gerahs: twenty shekels, five and twenty shekels, fifteen shekels, shall be your maneh." Ezek 45:10-12
- "And this is the fashion which thou shalt make it of: The length of the ark shall be three hundred cubits, the breadth of it fifty cubits, and the height of it thirty cubits." Gen 6:15

There are dozens of other examples. It seems that sometimes our ability to measure matters to God. In particular, if we study science we must be able to do accurate measurement.

I. MATHEMATICS - THE LANGUAGE OF SCIENCE.

Visual #2-1

As we "do science" in order to gain new knowledge, we automatically make observations. For example, we might note that if we mix two chemicals they get hot. This is *qualitative* data. Others should be able to try to repeat our results as closely as possible in order in order to confirm or refute our conclusions. To make it easier to get precise measurements, it is good to report our observations as numeric (*quantitative*) data, e.g., mixing the two chemicals results in a temperature increase of 15.0 degrees Celsius. Like it or not, experimentation requires math.

Math has been with us since prehistoric times.

Arithmetic.

Visual #2-2

- It seems likely that it began with someone inventing a system of counting.
- Some unknown person probably later realized that addition and subtraction would be faster than counting, and that multiplying and dividing would be faster than adding and subtracting.
- Fractions, a form of division, have been in use since prehistoric days.
- Later enhancements to math included raising numbers to powers or taking roots of numbers. Logarithms and exponents fall into this category. These are faster versions of multiplying and dividing.

Algebra.

Visual #2-3

- Along the way, ancient mathematicians such as Diophantus of Alexandria (3rd century A.D.) realized that mathematics needed to be able to deal with unknown quantities, which could be represented by *variables*. This concept was greatly advanced by Arab mathematicians, who gave us the term Algebra (from al-jabr, a reunion of broken parts). Algebra is used extensively in the physical sciences.

Geometry.

- One of the important ancient forms of mathematics was geometry, from the Greek for "earth measurement." By using shapes and angles, mathematicians were able to determine areas and volumes, both of which are very important in modern science. They were also useful to rulers who wanted to collect as much tax as possible from property owners. (See Gen. 47:19-20.)

Though we can speculate, nobody knows for sure why ancient mathematicians decided to break a circle down into 360 degrees instead of some other number.

Visual #2-4

> Some critics claim that the author of 2 Chronicles 4:2 did not know that the circumference of a circle is about 3.14 times its diameter (represented by the Greek letter π) when he wrote
>
> "Also he made a molten sea of ten cubits from brim to brim, round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it round about."
>
> Critics say that if the diameter was 10 cubits, the circumference should have been 31.4. Therefore, the Bible must be wrong. All one has to do is read three verses further to see how ridiculous the argument is. Verse 5 says that the thickness of the basin was a handbreadth. The diameter given was the outside measurement; the circumference was the inner measurement. The difference was the thickness of the basin. Though the outer circumference would have been about 31.4 cubits, 30 cubits is a perfectly reasonable value for the inner measurement.

Trigonometry.

Visual #2-5

- The study of triangles led to trigonometry, which is extensively used in physics.

Statistics.

- In the decades before his death in 1601, the Danish astronomer Tycho Brahe collected enormous amounts of data regarding the positions of various heavenly bodies at different times. His assistant, mathematician Johannes Kepler, applied statistical analysis to this vast quantity of numbers to discover his Laws of Planetary Motion, still in use over 400 years later.

Graphing.

Visual #2-6

- René Descartes (17^{th} century) invented the Cartesian coordinate system with perpendicular axes that we still use for graphing.
- Mathematician William Playfair (18th - 19th century) is usually given credit for inventing line, bar, and circle/pie graphs. However, others such as Nicole Oresme (14^{th} century) may have predated him by several centuries. Whoever invented them, graphs make it much easier to visualize large quantities of numbers.

Calculus.

Visual #2-7

- In the late 1600s to early 1700s Gottfried Leibniz and Isaac Newton both worked on inventing calculus, though some think that the ancient mathematician and inventor Archimedes had done so thousands of years earlier.

  Isaac Newton revolutionized astronomy when he used differential calculus to analyze the motion of the planets. We could think of this version of calculus being used to break a complex mathematical shape down into an infinite number of infinitely small parts. By contrast, integral calculus puts together an infinite number of infinitely small parts into a complex shape.

Imaginary numbers.

Visual #2-8

- If you have studied algebra, you may be familiar with the *quadratic formula* used to solve for *x* in equations that go no higher than the second power, in the form $ax^2 + bx + c = 0$. The formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  However, this formula does not work for equations that involve numbers raised to the third power or higher. Mathematicians attempting to derive general formulas to solve equations with higher powers had to invent *imaginary* numbers such as the square root of -1 to solve them.
- Erwin Schrödinger revolutionized chemistry when he derived a wave equation to describe the behavior of matter in terms of waves. His equation and many others in both quantum chemistry and physics rely on imaginary numbers.

Though this course will be limited mainly to concepts and will include only a few equations, serious students of science will recognize the need to become proficient in the types of math relating to their preferred fields.

II. MEASUREMENT IN SCIENCE.

A. STANDARDS OF MEASUREMENT.

Suppose you lived a thousand years ago and needed some rope to tie up your horse. You would go to the local rope seller in your village and ask for rope. "How much?" he says. You reply, "One horse worth."

Visual #2-9

The problem is that not all horses are the same size. If your horse were small and you had to pay the same amount as someone with a large horse, you would feel cheated. In order to treat customers fairly, all the merchants in any particular area needed to use the same standard, probably set by the national or local government.

1. The English (Imperial) system.

Many systems such as the one used in the United States, the English or *Imperial* system, depended on body parts. It spread around the world due to English conquest.

- The yard was defined as the average length of a man's arm from the tip of the nose to the tip of the outstretched fingers.
- For most humans, the arm length is about three times the length of the foot.
- One of the stories for the origin of the inch is that three men would place their thumbs side by side, then take one third of the distance across the thumbs at the widest part – the "rule of thumb." Whether or not this is true, for most people the width of the thumb is about 1/12 the length of the foot.

Once the standard length was determined, an object of that size could be placed in a central location for anyone to copy.

2. The Metric System (SI).

Visual #2-10

The problem with systems based on body parts is that not everybody's parts have the same dimensions. Beginning around 1790, the French devised a standard that would be the same for anybody in the world. It was called the metric system (from the Greek word for measure, *metron*), now known by scientists as SI, for the *Systeme Internationale*. The only three countries in the world still using the English system are the U.S., Liberia, and Myanmar. Everybody else uses the metric system, especially scientists.

> Units of time such as the second are the same in the English and metric systems.

a. Length, area, volume, and mass.

Though the meter is now defined in terms of the distance light travels in a specific amount of time, it was originally based on the circumference of the earth as calculated by the Greek mathematician Eratosthenes (ca. 240 BC). The meter was defined as 1/10,000,000 of the distance from the North Pole to the Equator through Paris. (It was purely coincidental that this was not much different from the English yard.)

Visual #2-11

Visual #2-12

For smaller measurements, the meter was subdivided into tenths, hundreds, and so on. These are identified by prefixes such as milli-, centi-, and so on. For larger measurements, the meter was multiplied by ten, a hundred, and so on. The prefix kilo- indicates that the quantity is multiplied by a thousand.

The metric system made it easy to define other quantities as well.

- A square or cubic meter is a square or cube one meter long on each side.
- If we were to make a cube one centimeter on each side (a cubic centimeter, also known as a cm^3 or a cc) and fill it with pure water, the mass was defined as one gram.
- A box one decimeter on a side (a cubic decimeter) would contain 1,000 grams, or a kilogram. This volume is also called a liter.

b. Force, energy, pressure, and power.

- Acceleration is a change of velocity over time. The force required to accelerate an object of one kg at a rate increasing by 1 meter per second, e.g., from 2 m/s to 3 m/s to 4 m/s and so on, is defined as one *newton*.

  Acceleration could be written as a change in m/sec/sec, but is usually written as m/s^2. This indicates an increase in the rate of movement per second for every additional second. For instance, the acceleration due to gravity on earth is 9.8 m/s^2, indicating that the velocity of an object in free fall increases by 9.8 meters per second every second.
- A force of one newton applied over an area of one m^2 is defined as a *pascal*. This is a very tiny amount. Atmospheric pressure is measured in *kilopascals*.
- The amount of energy required to lift an object of mass 1 kg against gravity a distance of 1 meter is called a newton-meter, or a *joule*.
- Power has to do not just with the amount of energy transferred into or out of an object, but with how fast that energy is transferred. A transfer of 1 joule per second is called a *watt*.

c. Temperature and heat.

i. U.S. and its territories - Fahrenheit.

The U.S. and its territories are among the very few areas that still use the Fahrenheit temperature scale.

Visual #2-13

Temperature.

Dissolving various substances in water changes its freezing point. For instance, scattering salt on icy roads tends to melt the ice. However, there is a temperature at which water will freeze no matter how much salt is dissolved in it. When the German scientist Daniel Fahrenheit was developing his temperature scale in the early 1700s, he defined this temperature as zero degrees. He also used average human body temperature as a reference. On his scale, the freezing and boiling points of pure water worked out to 32° and 212°.

Heat energy.

The amount of heat energy required to raise the temperature of one pound of water (at its densest, 39.2°) by 1 degree Fahrenheit is called a British Thermal Unit, or BTU.

ii. The rest of the world - Celsius and Kelvin.

Temperature.

Swedish scientist Anders Celsius led the way in creating a different scale a few decades after Fahrenheit. Instead of basing it on water with different substances dissolved in it, he defined the freezing point of pure water as zero degrees and its boiling point as 100 degrees. Since this system makes the math involved in chemistry much easier, it was incorporated into the metric system.

Scientists also use an absolute temperature scale, named after the British scientist Lord Kelvin. This scale is commonly used in chemistry, in which the behavior of water is usually irrelevant. The Kelvin scale still divides the interval between the freezing and boiling points of water into 100 equal intervals (Kelvins rather than degrees), but it defines absolute zero as the point at which all molecular motion would theoretically stop. This would be about -273.16 on the Celsius scale.

The math involved in using the Fahrenheit scale in chemistry requires some complicated conversions, whereas it is easy to convert between Celsius and Kelvin by adding or subtracting 273.16.

Heat energy.

In the metric system, one *calorie* is defined as the amount of heat energy necessary to raise the temperature of one gram of water (at its densest, about 4°) by 1 degree Celsius. The amount needed to raise the temperature of one kg of water is a kilocalorie. The calories reported on food packages are actually kilocalories.

d. Measurements of other quantities.

Since everything goes by powers of 10, calculations involving other quantities such as magnetic flux and light intensity are also easier to do in the metric system.

B. EXPONENTIAL AND SCIENTIFIC NOTATION.

Visual #2-14

Scientists frequently have to deal with extremely large numbers such as the number of atoms in an object, the distance to stars, and so on. We can write these numbers much more easily by using exponents and scientific notation.

1. EXPONENTS.

Ten times one is 10, a one followed by one zero. Ten times ten is 100, a one with two zeroes. 10 x 10 x 10 is 1000, a one with three zeroes. 10 x 10 x 10 x 10 is ten thousand, a one with four zeroes. Each time we multiply by another ten, we add another zero.

A shorthand way to write the number of zeroes after the one is to write it as ten with an exponent, i.e., raised to a power that is the same as the number of zeroes. Thus, a trillion (1,000,000,000,000) is a one with twelve zeroes after it, which can be be written in exponential notation as 10^{12}. The exponent is the same as the number of zeroes following the numeral 1.

> A quirk of math that occurs because of the way exponents work in division problems is that ten to the zero power (10^0) is NOT equal to zero, but equals 1. In fact, anything to the zero power (except zero) equals one.

2. NEGATIVE EXPONENTS.

If we see a ten with a negative exponent, it does not indicate that the number itself is negative. Instead, the notation is used to indicate a fraction, often a very small one.

The practical effect of a negative exponent is simply to put a "1" on top of the original power of 10. For instance, 10^{-1} is the same as $\frac{1}{10^1}$, or 1/10; 10^{-2} is the same as $\frac{1}{10^2}$, or 1/100, and 10^{-6} is the same as $\frac{1}{10^6}$, or one over a million. This notation is common in measuring very small sizes such as the width of an atom, which is on the order of 10^{-10} meters.

3. SCIENTIFIC NOTATION.

We could express the number of atoms in a lead fishing sinker as about 150,000,000,000,000,000,000,000 (a hundred fifty sextillion), but this would be very cumbersome to write. Instead, scientists use scientific notation. This system uses one digit before a decimal point, perhaps some other digits after the decimal, and ten to a power showing how many zeroes there would be if the number were written out showing only the first digit. For instance, in scientific notation the above number would be written as 1.5×10^{23}. The approximate distance to the sun, 93 million miles, would be written as 9.3×10^{7} miles.

Note that the exponent is not necessarily the number of zeroes that would be present after the last nonzero digit. For instance, the distance to the sun could be written as 93 x 10^6 miles, but this is not correct scientific notion. In this system there is only one digit before the decimal point. So we move the decimal one place to the left, making the number 9.3 instead of 93. Since this number is now smaller by a factor of 10, we compensate by increasing the exponent by one, from 10^6 to 10^7.

C. PRECISION AND ACCURACY IN MEASUREMENT.

Visual #2-15

Scientific notation requires one digit before the decimal point and perhaps several digits after. The number of digits you should include depends on how precise your measurement is.

In ordinary conversation we often use accuracy and precision as synonyms. This is not the case in science, though. The two terms have distinct meanings. .

1. **ACCURACY** is how close your measurement is to the actual value. That is, ACCurate = close to ACtual. You know the actual value by the authority of someone who has very sophisticated and expensive equipment.
2. **PRECISION** is how closely you can repeat the measurement. That is, PREcise = REPeatable.

   The precision of your measurement depends largely on the quality of your measuring instrument.

   - For instance, if you are using a stick that does not have any markings on it but seems to be about a meter long, you could probably get a measurement that is precise to the nearest meter. You would only be able to report "1 meter." That is, to the nearest meter, the object you are measuring is a meter long.
   - If instead your measuring stick has markings indicating tenths of a meter, you could now report "1.0 meters," indicating that to the nearest tenth of a meter, the object is a meter long.
   - If your measuring stick has markings indicating hundredths of a meter (centimeters), you could now report "1.00 meters," indicating that to the nearest hundredth of a meter, the object is a meter long.
   - If your measuring stick has markings indicating thousandths of a meter (millimeters), you could now report "1.000 meters," indicating that to the nearest thousandth of a meter, the object is a meter long.

   In scientific notation, then, you would report your measurements using the different measuring sticks with different numbers of decimal places as 1 x 10^0, 1.0 x 10^0, 1.00 x 10^0, and 1.000 x 10^0. The more decimal places, the more precision you are claiming.

Visual #2-16

Remember from Chapter 1 that one of the ways we say we know things is by relying on authorities. The reason we use the mathematical and measurement rules that we do is because many authorities made them up over the centuries, and we decide to follow them.

CHAPTER 2 REVIEW QUESTIONS

1. Why does science require measurement? ____

____

2. What is the difference between qualitative and quantitative data? ______________________

____

3. What was often the basis of measurement systems in earlier centuries?

____

4. What object was used as the basis for measuring length in the original metric system as devised by the French? ____

5. The metric system identifies fractions or multiples of base units such as the meter by using _________________________.

6. The original definition of a gram was the mass of 1 cm^3 of ______________________ .

7. Acceleration is a change of _______________________ over time.

8. On the Fahrenheit temperature scale, _______________ was defined as the temperature at which water would freeze no matter how much of a substance was dissolved in it.

9. On the Celsius scale, zero was defined as the temperature at which _________________ water would freeze.

10. What is the name of the absolute temperature scale used by chemists? __________________

11. How much is 10^0 ? __________

12. Scientific notation is used to make it easier to write very ____________________ or very __________________ numbers.

13. Write the number twelve million in scientific notation. ________________________

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